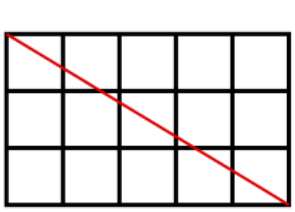
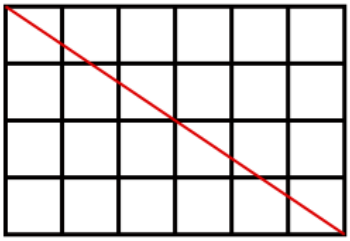
Puzzle of the Week
Crossing Lines

These two rectangles have been cut in half using a red line.

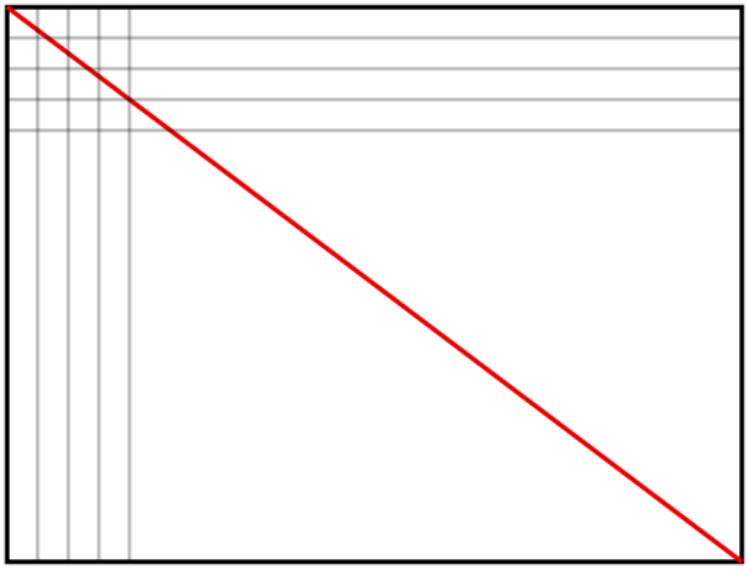
In the 3 by 5 rectangle: the red line crosses 6 sides of small squares. Check to see you agree!

In the 4 by 6 rectangle: the red line also crosses 6 sides of small squares. (We're not counting the place where the line goes through the corner of a square.)





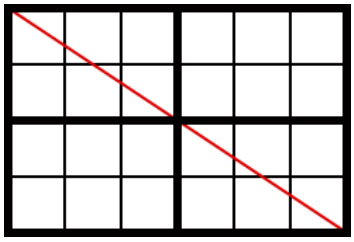
THE CHALLENGE: Here's a 36 by 42 rectangle that's been cut in half by a red line. How many sides of small squares will the red line cross?



EXPLORATION: What happens for rectangles of other sizes? What patterns do you see?

Puzzle of the Week
Crossing Lines – Notes

THE CHALLENGE & EXPLORATION: A diagonal will go through a corner of a box exactly when the change in height and width evenly divides the height and width of the rectangle..



Consider the 4 by 6 box in the introduction. Because 2 is a common factor of 4 and 6, we can break the larger 4 by 6 box into a 2 by 2 collection of 4/2 by 6/2 boxes. The diagonal of the 4 by 6 box will be the diagonal in 2 of the 2 by 3 boxes and go through the corner between them.

In the other introductory example of the 3 by 5 rectangle, the greatest common divisor is 1, so the diagonal does not go through any corners.

Suppose the rectangle is n by m, and that the greatest common divisor of n and m is c.

> **c = 1:** In this case, the diagonal will cross all n - 1 internal horizontal lines and all m - 1 internal vertical lines. So, it will cross a total of n + m - 2 box sides.
>
> **c > 1:** Break the diagonal into c pieces. Each piece of the diagonal will be a diagonal for a n/c by m/c rectangle, where the greatest common factor of n/c and m/c is 1. So, each piece of the diagonal will cross n/c + m/c - 2 box sides. All the pieces together will cross c x (n/c + m/c - 2) = n + m - 2c box sides.

The general answer is n + m - 2c. Let's apply this to the two introductory examples and the challenge puzzle.

For a 3 by 5 rectangle, c will be 1 and the answer is 3 + 5 - (2 x 1) = 8 – 2 = 6.

For a 4 by 6 rectangle, c will be 2 and the answer is 4 + 6 - (2 x 2) = 10 - 4 = 6.

For the challenge 36 by 42 rectangle, c will be 6 and the answer is 36 + 42 - (2 x 6) = 78 - 12 = 66.