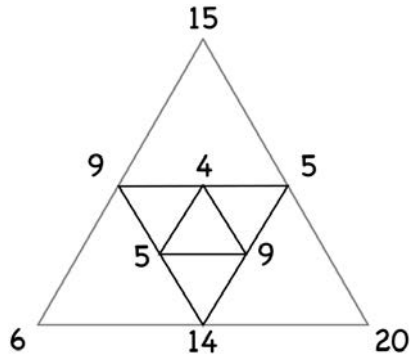


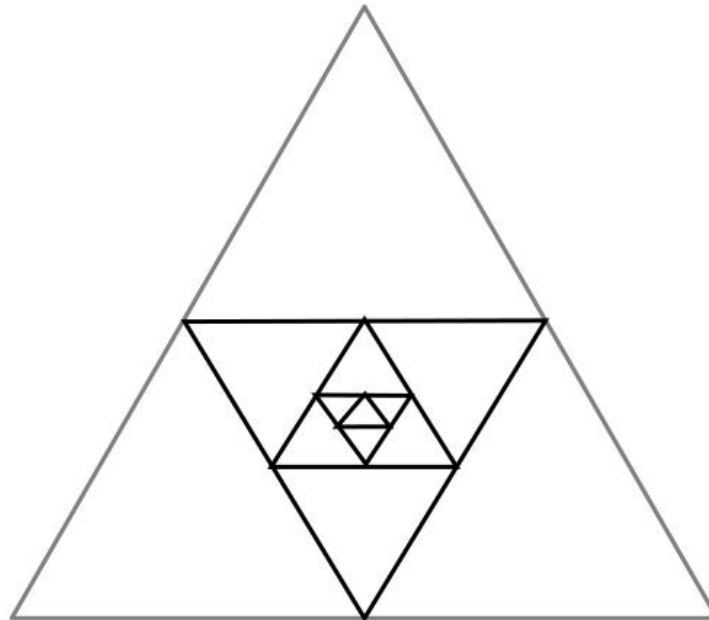
Puzzle of the Week

Difference Triangles

Start by putting numbers in the corners of a triangle. Then, put the differences of those numbers in the middle of the sides and make a new triangle with those differences. Continue this process and see where it takes you.



THE CHALLENGE: 1) Are there starting numbers for which this will go on forever without repeating, or does it always settle down into a simple pattern eventually? 2) If there is a simple ending pattern, what is it? 3) For which starting numbers will it end up with three 0's?



Puzzle of the Week

Difference Triangles – Notes

THE CHALLENGE: First, just play with it and see what happens from various groups of starting numbers. Look for patterns and enjoy the experimentation. Here are some observations you can make.

Observation 1: If the numbers are all positive, then the maximum number will decrease with each round.

Because the difference between two positive numbers is less than the larger of them, the maximum number must decrease.

Observations 2 - 4 are easier to try yourself than to read about why they are true.

Observation 2: If one of the numbers is 0 and the other two numbers are positive and different, the next round will have three positive numbers and the maximum number will be the same as the previous round.

Observation 3: If there are two 0's and a positive number, the next round will have that positive number twice and the third number will be 0.

Observation 4: If one of the numbers is 0 and the other two numbers are the same, all the rounds after that will be identical to that.

So, putting these observations together, the maximum number in a round will decrease if there are no 0's, and will stay the same for one round if there is a single or a double 0. That means that no matter what you start with, you will eventually reach a round with two or three 0's, and from then on it will not change.

Observation 5: The only way to end up with three 0's is to start with three numbers that are the same and get to the three 0's in one round..

The first time three 0's occur, the previous round must have had three identical positive numbers. If you play with it briefly, you will see there is no way to produce three identical positive numbers by using differences from an earlier round.