

Puzzle of the Week

Egyptian Fractions – 1

Around 4000 years ago, the ancient Egyptians developed a special way of writing fractions. **Unit Fractions**, which are fractions with 1 in the numerator such as $\frac{1}{3}$ and $\frac{1}{8}$, were important to them, and are also known as **Egyptian Fractions**. The Egyptians wrote *any* fractional quantity as an **Egyptian Fraction Sum**, which is a sum of Egyptian Fractions with no duplicates. For example, for $\frac{7}{8}$ they wrote the Egyptian Fraction Sum $\frac{7}{8} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.

THE CHALLENGE: Write 1 as an Egyptian Fraction Sum using as few fractions as possible.

$$1 = 1/A + 1/B + \dots$$

EXPLORATION: Convince yourself that you can't use fewer fractions to get 1. Write $\frac{2}{3}$, $\frac{3}{2}$, $\frac{2}{7}$, $\frac{3}{7}$, and $\frac{4}{7}$ as an Egyptian Fraction Sum. Play with other numbers and look for patterns for ways to break a fraction into Egyptian Fractions that will help you work with them.

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Egyptian Fractions – 1 – Notes

THE CHALLENGE: The answer for this is simple enough: $1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$.

EXPLORATION: The only way to use two fractions would be as $1 = \frac{1}{2} + \frac{1}{2}$, and that is not allowed.

A **Greedy Algorithm** can be used for creating Egyptian Fraction Sums. At each step, this algorithm picks the unit fraction with the smallest possible denominator (the largest possible value for the fraction). Using that approach, $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$; $\frac{2}{7} = \frac{1}{4} + \frac{1}{28}$; $\frac{3}{7} = \frac{1}{3} + \frac{1}{11} + \frac{1}{231}$; and $\frac{4}{7} = \frac{1}{2} + \frac{1}{14}$. The Greedy Algorithm can lead to some needlessly large denominators, but it always works. For example, $\frac{3}{7} = \frac{1}{3} + \frac{1}{11} + \frac{1}{231}$, but it can be written more simply as $\frac{3}{7} = \frac{1}{4} + \frac{1}{7} + \frac{1}{28}$.

One way to do $\frac{3}{2}$ would be to start with $1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$, and combine that with $\frac{1}{2} = \frac{1}{4} + \frac{1}{6} + \frac{1}{20}$. Doing this, we end up with $\frac{3}{2} = 1 + \frac{1}{2} = (\frac{1}{2} + \frac{1}{3} + \frac{1}{6}) + (\frac{1}{4} + \frac{1}{6} + \frac{1}{20})$. Your students may find other interesting ways to do this.

If you go to Wikipedia, you will find many interesting formulas for Egyptian Fraction Sums. One useful formula for $\frac{1}{n}$ is $\frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)}$. For example, $\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$.