

Puzzle of the Week

Egyptian Fractions – 2

Around 4000 years ago, the ancient Egyptians developed a special way of writing fractions. **Unit Fractions**, which are fractions with 1 in the numerator such as $\frac{1}{3}$ and $\frac{1}{8}$, were important to them, and are also known as **Egyptian Fractions**. The Egyptians wrote *any* fractional quantity as an **Egyptian Fraction Sum**, which is a sum of Egyptian Fractions with no duplicates. For example, for $\frac{7}{8}$ they wrote the Egyptian Fraction Sum $\frac{7}{8} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.

THE CHALLENGE: Write $\frac{39}{50}$ as an Egyptian Fraction Sum using as few fractions as possible.

$$\frac{39}{50} = \frac{1}{A} + \frac{1}{B} + \dots$$

EXPLORATION: Is your answer the absolute best? If yes, what are your reasons?



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Egyptian Fractions – 2 – Notes

THE CHALLENGE: You can chip away at this by subtracting the largest Egyptian Fraction you can and see what is left over. This always works with any fraction, but it doesn't always produce the fewest number of Egyptian Fractions.

First, $39/50 > 1/2$, so $39/50 = 1/2 + 14/50 = 1/2 + 7/25$.

$7/25 > 1/4$, so subtract $1/4$ off next. $39/50 = 1/2 + 1/4 + 3/100$.

Finally, $3/100 = 2/100 + 1/100 = 1/50 + 1/100$.

Putting it all together, $39/50 = 1/2 + 1/4 + 1/50 + 1/100$.

EXPLORATION: I am not aware of a shortcut for analyzing whether fewer fractions are possible.

Consider if it were possible to use three fractions. The fraction with the largest value must be at least $1/3$. Otherwise, the largest value for the sum of the three fractions would be $1/4 + 1/5 + 1/6$, and that is not big enough.

- The largest fraction is $1/2$: The remaining value is $39/50 - 1/2 = 7/25$. The larger of the two remaining fractions must be at least $1/7$. That leads to four possibilities to look at: $1/2 + 1/4$, $1/2 + 1/5$, $1/2 + 1/6$, and $1/2 + 1/7$.
- The largest fraction is $1/3$: The remaining value is $39/50 - 1/3 = 67/150$. The larger of the two remaining fractions must be at least $1/6$. That leads to two possibilities to look at: $1/3 + 1/4$ and $1/3 + 1/5$.

It's not pretty. However, there are not that many combinations to check to see that three fractions will not work.