

Puzzle of the Week

Egyptian Fractions – 3

Around 4000 years ago, the ancient Egyptians developed a special way of writing fractions. **Unit Fractions**, which are fractions with 1 in the numerator such as $\frac{1}{3}$ and $\frac{1}{8}$, were important to them, and are also known as **Egyptian Fractions**. The Egyptians wrote *any* fractional quantity as an **Egyptian Fraction Sum**, which is a sum of Egyptian Fractions with no duplicates. For example, for $\frac{7}{8}$ they wrote the Egyptian Fraction Sum $\frac{7}{8} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.

THE CHALLENGE: Write each of the fractions $\frac{1}{20}$, $\frac{2}{20}$, $\frac{3}{20}$, $\frac{4}{20}$, $\frac{5}{20}$, $\frac{6}{20}$, $\frac{7}{20}$, $\frac{8}{20}$, $\frac{9}{20}$, and $\frac{10}{20}$ as an Egyptian Fraction or an Egyptian Fraction Sum using only two fractions. If the fraction is already an Egyptian Fraction, such as $\frac{1}{20}$, you can use it as is.

$$X/20 = 1/A$$

or

$$X/20 = 1/A + 1/B$$

EXPLORATION: Play around with other groups of fractions with the same denominator (e.g. 6, 7, 8, 12, 17, 21, 50, etc.). For which denominators do you need to create Egyptian Fraction Sums with more than two fractions?

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Egyptian Fractions – 3 – Notes

THE CHALLENGE: Here is the list of results:

- $1/20 = 1/20$
- $2/20 = 1/10$
- $3/20 = 2/20 + 1/20 = 1/10 + 1/20$
- $4/20 = 1/5$
- $5/20 = 1/4$
- $6/20 = 3/10 = 2/10 + 1/10 = 1/5 + 1/10$
- $7/20 = 5/20 + 2/20 = 1/4 + 1/10$
- $8/20 = 2/5 = 1/3 + 1/15$
- $9/20 = 5/20 + 4/20 = 1/4 + 1/5$
- $10/20 = 1/2$

EXPLORATION: Here are a few more families of fractions. The ones whose denominators create fractions that are more likely to reduce are easier to do. In particular, denominators that are primes tend to be messy.

- $1/6 = 1/6$
- $2/6 = 1/3$
- $3/6 = 1/2$
- $4/6 = 2/3 = 1/2 + 1/6$
- $5/6 = 1/2 + 1/3$

- $1/7 = 1/7$
- $2/7 = 1/4 + 1/28$
- $3/7 = 1/4 + 5/28 = 1/4 + 1/7 + 1/28$
- $4/7 = 1/2 + 1/14$
- $5/7 = 1/2 + 3/14 = 1/2 + 1/7 + 1/14$
- $6/7 = 1/2 + 5/14 = 1/2 + 1/3 + 1/42$

- $1/8 = 1/8$
- $2/8 = 1/4$
- $3/8 = 1/4 + 1/8$
- $4/8 = 1/2$
- $5/8 = 1/2 + 1/8$
- $6/8 = 3/4 = 1/2 + 1/4$
- $7/8 = 1/2 + 1/4 + 1/8$