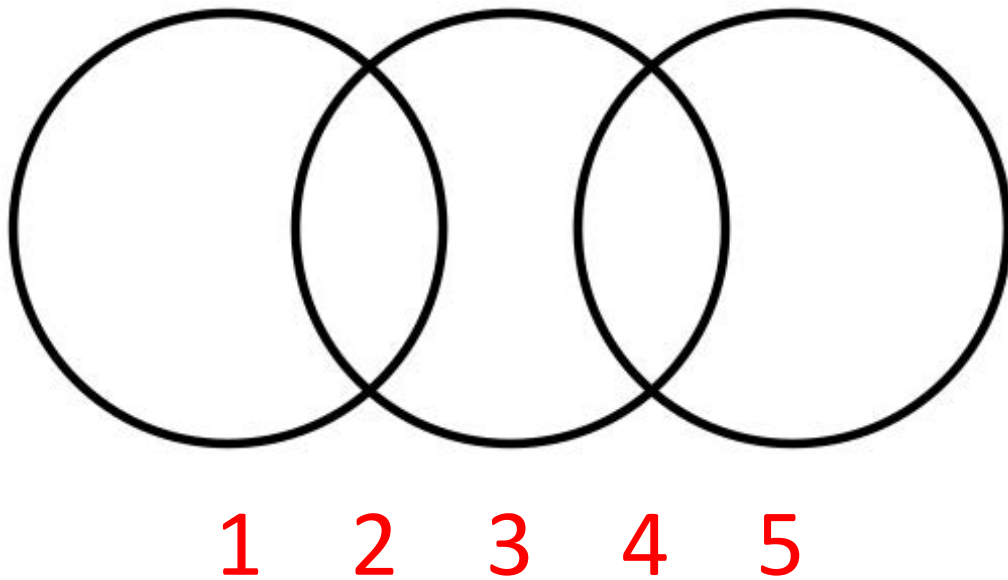


Puzzle of the Week

Equal Sums – 1

THE CHALLENGE: Here is a diagram created by overlapping three circles. The overlapping circles create five regions. Put a number in each of the five regions, using each of the numbers 1 to 5 exactly once, so that the sum of the numbers in each circle is the same.

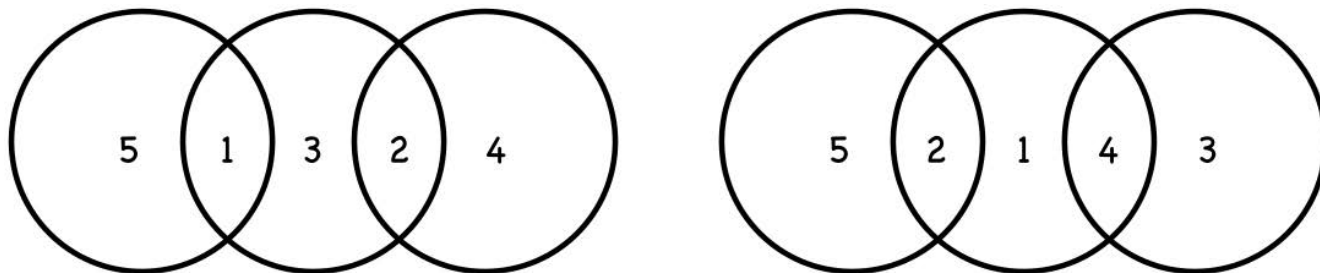


EXPLORATION: How many different answers can you find? How do you know if you have found them all?

Puzzle of the Week

Equal Sums – 1 – Notes

THE CHALLENGE & EXPLORATION: There are two solutions. Going from left to right, the sum in each circle is 6 and 7.



Analyze the possibilities by letting A and B be the two numbers in the intersections of the circles. Let Sum be the common sum inside each circle. Then $3 \times \text{Sum} = 1 + 2 + 3 + 4 + 5 + A + B = 15 + A + B$.

The left side of $3 \times \text{Sum} = 15 + A + B$ is a multiple of 3, so the right side is as well. This forces $A + B$ to be a multiple of three. That leaves only three possibilities.

- $A + B = 3$. In this case $3 \times \text{Sum} = 15 + 3 = 18$ tells us $\text{Sum} = 6$, and A and B are 1 and 2.
- $A + B = 6$. In this case $3 \times \text{Sum} = 15 + 6 = 21$ tells us $\text{Sum} = 7$. $A + B = 6$ forces A and B to be either 1 and 5 or 2 and 4. Having A and B be 1 and 5 does not work (1 is repeated), so that leaves us with just 2 and 4.
- $A + B = 9$. In this case $3 \times \text{Sum} = 15 + 9 = 24$ tells us $\text{Sum} = 8$, and A and B are 4 and 5. However, because A and B are both in the middle circle, it is not possible for $A + B = 9$ and yet the Sum is only 8. So this case cannot happen.