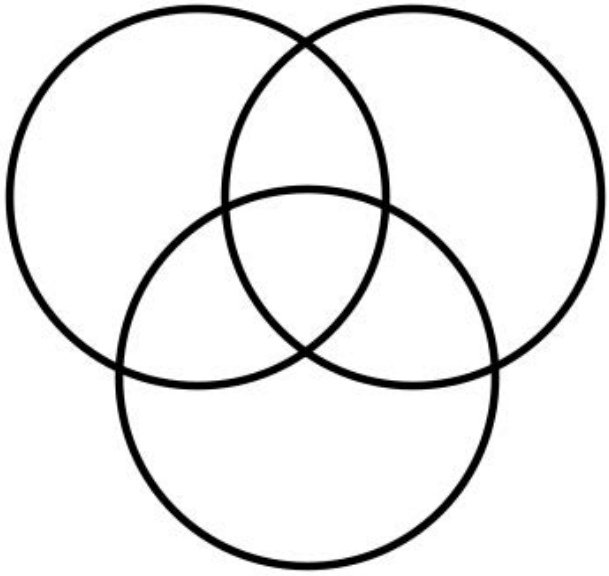
Puzzle of the Week
Equal Sums – 3

THE CHALLENGE: Here is a diagram created by overlapping three circles. The overlapping circles create seven regions. Put a number in each of the seven regions, using each of the numbers 1 to 7 exactly once, so that the sum of the numbers in each circle is the same.



1 2 3 4 5 6 7

EXPLORATION: How many different answers can you find? How do you know you have found them all?

Puzzle of the Week
Equal Sums – 3 – Notes

THE CHALLENGE & EXPLORATION: The first question with these problems is what the possible sums in the circles are. Narrow this down by looking at the sum of the entries in two ways. Let A, B, and C be the three values where two circles intersect, let M be the value where all three circles intersect, and let Sum be the common sum in each circle. The sum of the three circles is 3 x Sum. That sum is also the sum of the numbers from 1 to 7 plus A + B + C plus 2 x M. Therefore, 3 x Sum = 28 + A + B + C + 2 x M.

Before going further with this, note that if we start with any solution, we can get a new solution by subtracting all the entries from 8. This will have the effect of replacing Sum by 32 - Sum. Because of this, we only need to look for values of Sum up to 16 - the remaining larger values can be obtained by subtracting those solution entries from 8.

The smallest A + B + C + 2 x M can be is 2 + 3 + 4 + 2 x 1 = 11. So the smallest Sum can be is (28 + 11) / 3 = 13. Consequently, we want to see which of the Sum values from 13 to 16 are possible. It turns out that they are all possible.

Here is one solution for each Sum value starting at 13 and ending at 16. There are many more.