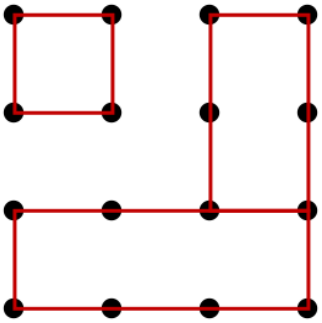
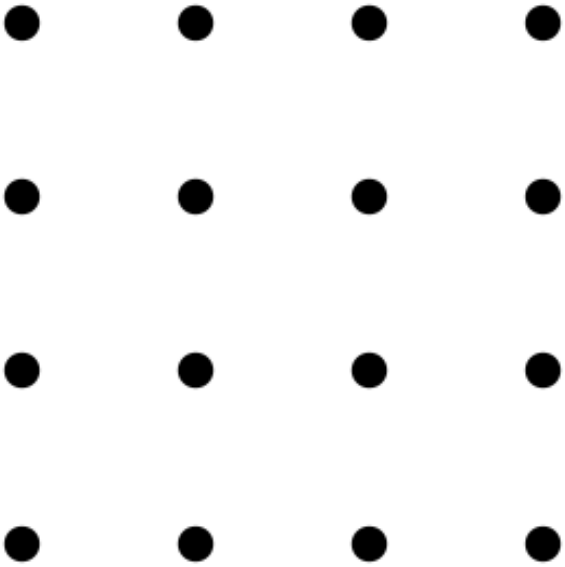
Puzzle of the Week

# *Finding Rectangles in Squares*

Drawn in red in this grid are a 1 by 1 rectangle (square), a 2 by 1 rectangle and a 1 by 3 rectangle.



THE CHALLENGE: Count the number of all the rectangles you can make in this 4 by 4 grid.



EXPLORATION: Calculate the number of rectangles in a 2 by 2 grid and a 3 by 3 grid. Compare those answers to your answer for the 4 by 4 grid. What is the pattern you see, and can you explain it?

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# *Finding Rectangles in Squares – Notes*

THE CHALLENGE & EXPLORATION: The careful student will find 36 rectangles. The key is to be organized in your approach. Here is a count for each type of rectangle:

- 1 x 1 - 9
- 1 x 2 - 6
- 1 x 3 - 3
- 2 x 1 - 6
- 2 x 2 - 4
- 2 x 3 - 2
- 3 x 1 - 3
- 3 x 2 - 2
- 3 x 3 - 1

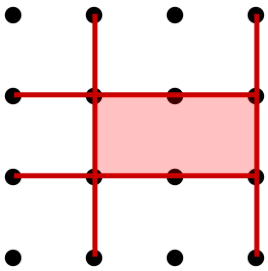
Adding these all together, we get 9 + 6 + 3 + 6 + 4 + 2 + 3 + 2 + 1 = 36 rectangles.

For n by n grids, the number of rectangles is:

- 2 by 2 - 1 = $1^2$
- 3 by 3 - 9 = $3^2$
- 4 by 4 - 36 = $6^2$
- 5 by 5 - 100 = $10^2$
- 6 by 6 - 225 = $15^2$

They are all squares. What are the numbers 1, 3, 6, 10, and 15? They are the triangular numbers, and they are also the number of ways of choosing two things from n things.

To see why this works, consider how a rectangle is determined inside the square. As shown in this illustration, every rectangle is determined by picking the two vertical sides and the two horizontal sides. And for each choice of positions for the sides, there is one rectangle that is determined.



So, the number of rectangles is equal to the number of ways of making these choices. In a 4 by 4 grid, there are 4 vertical lines to choose from and 4 horizontal lines to choose from. The number of ways of choosing 2 things from 4 things is 6. So the number of rectangles is 6 x 6 = 36.

The same reasoning is true for other sizes of square grids.