

Puzzle of the Week

Fractions – 11

THE CHALLENGE: Use the numbers 1 to 9 at most once each in these boxes. Put the numerators and denominators in increasing order to keep your solutions orderly.

$$\frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} = 1$$

1 2 3 4 5 6 7 8 9

Puzzle of the Week

Fractions – 11 – Notes

THE CHALLENGE: Write the equation with variables to make it easier to talk about: $A/B \times C/D \times E/F = 1$.
Similar to the Fractions – 9 puzzle notes, multiply by the denominators to make this $A \times C \times E = B \times D \times F$.

As there is only one 5 and one 7, they cannot be put into this equation. Thinking in terms of primes helps a lot. We need to balance the number of 2's and 3's on both sides of this equation using the remaining seven numbers: 1, 2, 3, 4, 6, 8, and 9 to fill the six slots.

The 3's are more limited, so there are only two possibilities when solving $A \times C \times E = B \times D \times F$.

- 3 is on the left side and 6 is on the right side. We need to use all four of the remaining numbers. This is impossible to do. Including 6, there are seven factors of 2 in all the numbers - one 2 for 2 and 6, two 2's for the 4, and three 2's for the 8. So, it is impossible to split these seven things (2's) evenly between the two sides.
- 9 is on the left side and 3 and 6 are on the right side. We will be leaving out one of the remaining four numbers to complete the equation. We have seven 2's to deal with, and leaving out one number must give an even number of 2's - we can either leave out the 2 or the 8.
 - $9 \times 1 \times 8 = 3 \times 6 \times 4$
 - $9 \times 1 \times 4 = 3 \times 6 \times 2$

So, up to switching denominators around, we have four solutions (where the second two are just the flip of the first two):

- $(1 \times 8 \times 9) / (3 \times 4 \times 6)$
- $(1 \times 4 \times 9) / (2 \times 3 \times 6)$
- $(3 \times 4 \times 6) / (1 \times 8 \times 9)$
- $(2 \times 3 \times 6) / (1 \times 4 \times 9)$