

Puzzle of the Week

Fractions – 5

Example: $2/5$ is less than $1/2$, $19/38$ equals $1/2$, and $4/7$ is larger than $1/2$. The difference between $4/7$ and $2/5$ is $(4/7 - 1/2) + (1/2 - 2/5) = 1/14 + 1/10$, which is not particularly small. The fractions $2/5$, $19/38$, and $4/7$ use the digits 1 to 9 at most once each.

THE CHALLENGE: Use the numbers 1 to 9 at most once each to fill in these boxes. Make the difference between the largest and smallest fraction as small as possible.

$$\frac{\square}{\square} < \frac{1}{2}$$

$$\frac{\square\square}{\square\square} = \frac{1}{2}$$

$$\frac{\square}{\square} > \frac{1}{2}$$

1 2 3 4 5 6 7 8 9

Puzzle of the Week

Fractions – 5 – Notes

THE CHALLENGE: After some initial exploring, it is clear that $1/14 + 1/18$ is the best possible difference. That difference comes from using denominators of 7 and 9, and there are no better ones. The only ways to get that difference are: $5/9 - 3/7$ or $4/7 - 4/9$, and the second one is not allowed because four is repeated.

If 3, 5, 7, and 9 are used on the outside fractions, that only leaves 1, 2, 4, 6, and 8 to be used for the middle fraction. There are only two ways to get $1/2$ using those numbers without repeating a digit: $14/28$ and $41/82$.

This leaves two entries tied for first place:

$$3/7 < 14/28 = 1/2 < 5/9$$

$$3/7 < 41/82 = 1/2 < 5/9$$