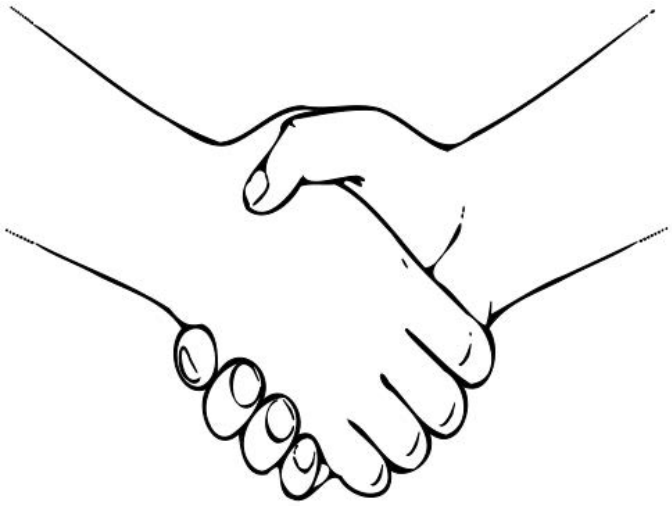
Puzzle of the Week

Handshakes at a Party – 1

Ten people were at a party. A lot of handshaking took place. When asked how many hands each person shook, the answers were: 3, 3, 4, 4, 4, 5, 5, 5, 6, and 8. When this was announced to the group, one person yelled out "That's impossible!"

THE CHALLENGE: Was that person right, did someone make a mistake in their handshake count? How do you know?



EXPLORATION: Look further into handshake counts and come up with a short list of counts that are possible and some that are impossible.

Puzzle of the Week

Handshakes at a Party – 1 – Notes

THE CHALLENGE: An interesting property of handshakes is that they are mutual - if I shake your hand, you are shaking my hand as well. This type of property can come up in other contexts, such as friendships that are mutual.

Because of this property of handshakes, if you total up the handshakes for all the people involved in an event, every handshake will be counted twice, once for each side of the handshake. Consequently, there should be an even number when all the handshakes are added up. If you add up 3, 3, 4, 4, 4, 5, 5, 5, 6, and 8, the total is 47, which is an odd number. So, a mistake must have been made!

EXPLORATION: From what was just discussed, if a collection of counts is going to be possible, their sum must be even. Another obvious requirement is that each individual count must be less than the total number of people (no one shakes their own hand).

However, those two conditions are not enough. For example, consider the counts 4, 4, 4, 4, 2. Of these five people, four of them have shaken everyone else's hand, so it is not possible for the last person to only have two handshakes. In a similar way, while 4, 4, 4, 3, 3 could happen, 4, 4, 4, 2, 2 cannot.

In "Handshakes at a Party - 2" we will see that the handshake list 7, 6, 5, 4, 3, 2, 1, 0 is impossible, despite having an even sum.

This leaves plenty of room to be explored further by the interested student.