Puzzle of the Week

Letter Substitutions – 14

THE CHALLENGE: Using each of the numbers from 1 to 9 exactly once, find the value for each of these letters that will make both of these equations true.

$$AB \times C = DE$$

$$F \times G = KL$$

EXPLORATION: Is there more than one solution?

Puzzle of the Week
Letter Substitutions – 14 – Notes

THE CHALLENGE & EXPLORATION: Start by narrowing down which letters might be 5.

Because 5 times anything will end in 5 or 0, neither of which is possible, 5 cannot be in the ones digits of any of the numbers. That only leaves A, D, and K as places for 5.

If A = 5, then AB is at least 51. C must be at least 2 (or AB would equal DE). In this case AB times C would be larger than a two-digit number. So, A cannot be 5.

For the remaining cases, we need to know the factorizations of all the numbers in the 50's, so here they are:

- 51 = 3 x 17 (repeats 1)
- 52 = 2 x 26 (repeats 2) = 4 x 13
- 53 = prime
- 54 = 2 x 27 (repeats 2) = 3 x 18 = 6 x 9
- 55 = 5 x 11 (repeats 1)
- 56 = 2 x 28 (repeats 2) = 4 x 14 (repeats 4) = 7 x 8
- 57 = 3 x 29
- 58 = 2 x 29 (repeats 2)
- 59 = prime

If D = 5, there are only three possibilities that don't repeat digits:

- 52 = 4 x 13 - This leaves 6, 7, 8, 9 for FGKL, which cannot work.
- 54 = 3 x 18 - This leaves 2, 6, 7, 9 for FGKL, which cannot work.
- 57 = 3 x 29 - This leaves 1, 4, 6, 8 for FGKL, which cannot work.

If K = 5, there are only two possibilities that don't repeat digits:

- 54 = 6 x 9 - This leaves 1, 2, 3, 7, 8 for ABCDE, and 27 x 3 = 81 !!!!
- 56 = 7 x 8 - This leaves 1, 2, 3, 4, 9 for ABCDE, which cannot work.

There is only one solution (ignoring swapping F and G) and it is:

27 x 3 = 81 and 6 x 9 = 54