Puzzle of the Week

Maximizing Products with 16

Look at ways to break up 16 into a sum of numbers that you can then multiply to get as big a product as possible. Writing 16 = 10 + 6 is a start, but 16 = 6 + 5 + 5 is better. Can you do better?

$$16 = 10 + 6 \text{ and } 10 \times 6 = 60$$

$$16 = 6 + 5 + 5 \text{ and } 6 \times 5 \times 5 = 150$$

THE CHALLENGE: What is the biggest product you can make by breaking 16 into a sum of numbers?

EXPLORATION: How does your strategy change if you replace 16 with 20, 50, or 100?

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Maximizing Products with 16 – Notes

THE CHALLENGE & EXPLORATION: Let's start by looking at some examples that replace 16 with smaller numbers. Getting this experience is almost always a good way to start, and it usually doesn't take long.

1 = 1 => 1

2 = 2 => 2

3 = 3 => 3

4 = 2 + 2 or just 4 => 4

5 = 2 + 3 => 2 x 3 = 6

6 = 3 + 3 => 3 x 3 = 9

7 = 2 + 2 + 3 => 2 x 2 x 3 = 12

8 = 2 + 3 + 3 => 2 x 3 x 3 = 18

9 = 3 + 3 + 3 => 3 x 3 x 3 = 27

10 = 2 + 2 + 3 + 3 => 2 x 2 x 3 x 3 = 36

As you look at these examples, a pattern emerges for breaking down larger numbers.

1. Any number larger than 4 should be replaced with smaller numbers!
2. Never use 1 unless you have no choice.
3. Use 2 + 2 instead of 4. They give the same result, but it is easier to see how to improve things when looking at 2's rather than 4's.
4. Always replace 2 + 2 + 2, which gives 2 x 2 x 2 = 8, with 3 + 3, which gives 3 x 3 = 9.

Using these rules, a general strategy emerges for attacking any number.

1. If the number is even, write it as a sum of 2's. If the number is odd, write it as 3 plus a sum of 2's.
2. Replace any group of three 2's with a group of two 3's.

With all this in place, we are ready to solve the problem.

16 = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 2 + 2 + 3 + 3 + 3 + 3, which gives a product of 2 x 2 x 3 x 3 x 3 x 3= 324.

As for the other numbers, we have:

20 = 10 x 2 = 2 + 3 x (3 x 2) = 2 + 3 x (2 x 3) = 2 + 6 x 3, which gives 2 x 3^6 = 2 x 729 = 1458.

50 = 25 x 2 = 2 + 8 x (3 x 2) = 2 + 8 x (2 x 3) = 2 + 16 x 3, which gives 2 x 3^16.

100 = 50 x 2 = 2 x 2 x 16 x (3 x 2) = 2 + 2 + 16 x (2 x 3) = 2 x 2 + 32 x 3, which gives 4 x 3^32.