Puzzle of the Week

Moving Digits – 1

Any multiple of 10 has the property that if we remove the ones digit, the new number evenly divides the original number. Take 50 for example. If you remove the 0 you get 5, and 5 evenly divides 50.

THE CHALLENGE: Of the two-digit numbers that are not multiples of 10, which ones have the property that if their ones digit is deleted, the new number evenly divides the original number?

When does A divide AB?

EXPLORATION: Other than multiples of 10, investigate why there are no numbers larger than 100 that have this property.

Puzzle of the Week
Moving Digits – 1 – Notes

THE CHALLENGE: It is straightforward to simply list all the numbers: 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 24, 26, 28, 33, 36, 39, 44, 48, 55, 66, 77, 88, and 99.

EXPLORATION: What follows is some formal algebra to justify why it is impossible. Your students may come up with reasoning that involves numbers and examples that use reasoning equivalent to this algebra, and that would be wonderful!

For example, a student might look at a number such as 127. If you divide 127 by 12, you get 10 + 7/12. After doing several such examples, they can reasonably claim that it can never work for numbers larger than 100.

Algebra: Write the number as 10a + b, where b is the ones digit (we are assuming b is not 0). We want to find numbers where a evenly divides 10a + b. We are assuming that a evenly divides 10a + b. So, if we divide 10a + b by a, we get 10 + b/a, and a must evenly divide b. Because b is between 1 and 9, that forces a to be less than 10 and we are done.