Puzzle of the Week

Moving Digits – 2

THE CHALLENGE: Find a 4-digit number, ABCD, that satisfies this interesting equation when you reverse the digits:

$$ABCD \times 4 = DCBA$$

EXPLORATION: Investigate why this cannot happen for numbers less than 1000. Also, look for numbers larger than 9,999 that have this property.

Puzzle of the Week
Moving Digits – 2 – Notes

THE CHALLENGE: Because multiplying ABCD by 4 produces a 4-digit number, A must be 1 or 2. Also, because the result of multiplying D by 4 has a ones digit of A, we know A must be even. So A = 2.

Our equation is now 2BCD x 4 = DCB2.

D x 4 produces a 2 means that D is 3 or 8. Notice that 4 times a number larger than 2000 creates a number that is at least 8000. Hence, D must be 8.

Our equation is now 2BC8 x 4 = 8CB2.

B must be less than 3, or 2B x 4 will be bigger than 8999. Looking at C8 x 4 and going through the ten values of C, the only way to get a value of B in that range is if C is 2 or 7 and B is 1. Therefore, we only have two numbers to check: 2128 or 2178.

The answer is 2178!

EXPLORATION: The logic that shows that the number must have a high-order digit of 2 and a low-order digit of 8 holds no matter how many digits the number has.

Looking at two-digit numbers, 28 does not work. For three-digit numbers, 2x8 does not work for any value of x.

The analysis for larger numbers is perhaps more than you'd like to read. Here are the next few numbers: 21978, 219978, and 2199978.