

Puzzle of the Week

Moving Digits – 3

THE CHALLENGE: Find a 4-digit number, ABCD, that satisfies this interesting equation when you reverse the digits:

$$ABCD \times 9 = DCBA$$

EXPLORATION: Investigate why this cannot happen for numbers less than 1000. Also, look for numbers larger than 9,999 that have this property.

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Moving Digits – 3 – Notes

THE CHALLENGE: Because multiplying ABCD by 9 produces a 4-digit number, A must be 1.

Our equation is now $1BCD \times 9 = DCB1$.

D x 9 produces a 1 means that D must be 9.

Our equation is now $1BC9 \times 9 = 9CB1$.

B must be less than 2 so that $1BC9 \times 9$ will be less than 9999. $C9 \times 9$ ends in B1 leaves only two possibilities for C - either C is 8 (B = 0) or 7 (B = 1). This gives two possible numbers: 1089 or 1179. Only 1089 works.

The answer is 1089.

EXPLORATION: No matter the number of digits, the logic holds that the high-order digit should be 1 and the low-order digit should be 9.

For two-digit numbers, 19 does not work. For three-digit numbers, numbers of the form $1x9$ do not work.

The analysis for larger numbers is perhaps more than you'd like to read. Here are the next few numbers beyond 9999: 10989, 109989, and 1099989.