Puzzle of the Week

Moving Digits – 4

THE CHALLENGE: Find a six-digit number whose product by 2, 3, 4, 5, and 6 results in numbers that use the same six digits.

EXPLORATION: If you have worked with decimal values for fractions, do the six products from The Challenge remind you of any fractions?

Puzzle of the Week

Moving Digits – 4 – Notes

THE CHALLENGE: Represent x, the six-digit number as ABCDEF.

A = 1: Because 6 x ABCDEF is a six-digit number, A must be 1. Any other value would make 6x bigger than a six-digit number.

A, B, C, D, E, F are all different: Multiply ABCDEF by 1 through 6 with results between 100000 and 999999 forces the high-order digits of 1x, 2x, 3x, 4x, 5x, and 6x to start at 1 and end no higher than 9, and therefore they must all be different.

None of A, B, C, D, E, and F is 0: Each one of these digits takes a turn at being the high-order digit, so they can never be 0.

F is odd and not 5: If F were even, then 5x would end in 0. If F = 5, then 2x would end in 0.

Each of A, B, C, D, E, and F takes a turn at being a ones digit: A consequence of F being odd and not equal to 5 is that the ones digits of the products 1x, 2x, 3x, 4x, 5x, and 6x are all different. Therefore, the ones digits must take on the six values for A through F.

F = 7: 1 is one of the values, so one of the multiples has 1 as a ones digits. The only multiple where that can happen is 3x. Because 3x has a ones digit of 1, F must be 7.

The ones digits in order are 7, 4, 1, 8, 5, and 2: This comes from looking at the products using F = 7.

The numbers have the form 1BCDE7, 2xxxx4, 4xxxx1, 5xxxx8, 7xxxx5, and 8xxxx2. This is essentially a summary of what we have so far.

B = 4: If B = 2, then 6x would be less than 800000. If B = 5, then 6x would be more than 899999.

The remaining digits are 2, 5, and 8. Some trial and error with filling in the remaining places ends up with the result.

The answer is: 142857.

EXPLORATION: This digit sequence is the same as the decimal representation for 1/7. If you look at the decimal representations for 2/7, 3/7, up to 6/7, each representation contains the same sequence of six numbers only starting at a different place in the sequence. Pretty amazing!