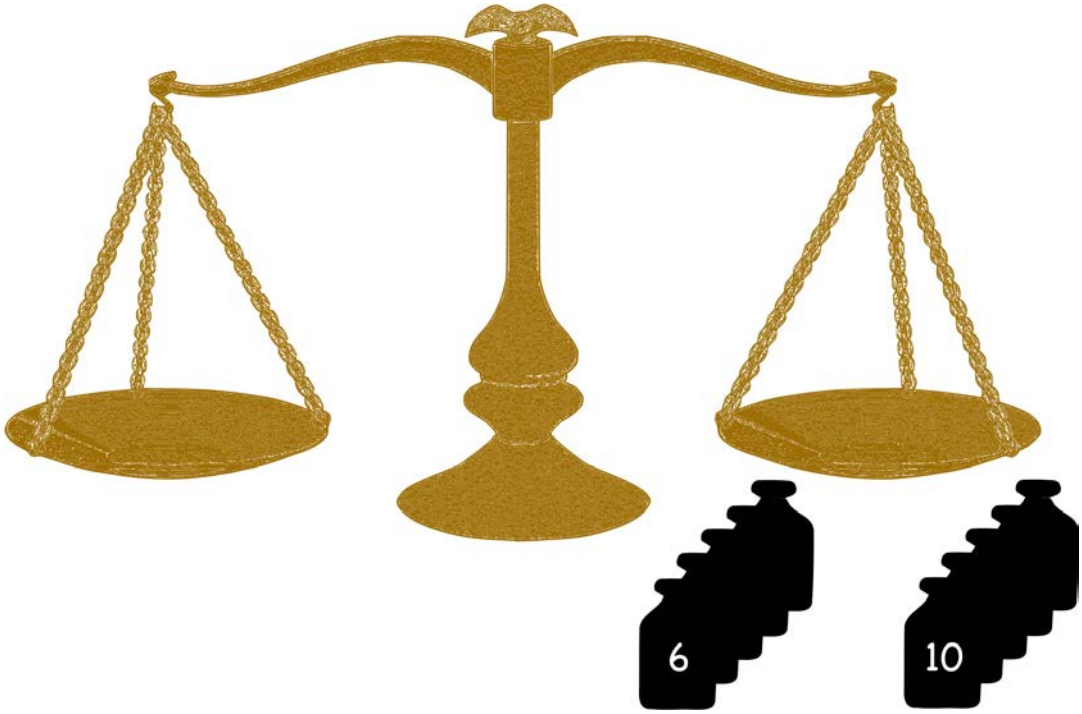
Puzzle of the Week
Pan Balance With Weights – 2

A pan balance tells you when its two sides are carrying the same amount of weight or whether one side is heavier than the other.

THE CHALLENGE: You have a very large collection of 6-ounce and 10-ounce weights to use on one side of a pan balance. By using two 6-ounce weights, you can measure a 12-ounce item. Which weights can you weigh exactly and which ones can't you weigh exactly?



EXPLORATION: How do your results change if you have 6- and 9-ounce weights? How about other pairs of weights that have a common divisor larger than 1? How do your results compare to the ones you got in "Pan Balance With Weights - 1"? Can you make use of that earlier work to save you reinventing things for this problem?

Puzzle of the Week

Pan Balance With Weights – 2 – Notes

THE CHALLENGE: The only difference between this puzzle and "Pan Balance With Weights - 1" is that the two numbers have a common divisor bigger than 1. In mathematics, we seek to take advantage of earlier work whenever we can.

For 6 and 10, the greatest common divisor is 2. Every multiple of either number will have a factor of 2, and so will all of their sums. One consequence of this is that any number that is not a multiple of 2 can never be weighed by our weights.

To take advantage of our earlier work, create a new weight - call it the TwoOunce. Now our weights are 3 TwoOunces and 5 TwoOunces. The advantage of doing that is that our numbers now have a greatest common divisor of 1, and we can use all of our earlier work. We can weigh all of the TwoOunce weights starting at (3 -1) x (5 - 1) TwoOunces, and we can weigh half of the TwoOunce weights up to that point.

Translating that result into ounces gives: We can weigh all the weights that are multiple of two ounces starting at 2 x 4 x 2 = 16 ounces, and half of the two-ounce multiples up to 16 ounces will be measurable.

EXPLORATION: For 6 and 9, the greatest common multiple is 3. So, only multiples of 3 ounces can possibly be hit, and all multiples of 3 ounces will be hit starting with 3 x (2 - 1) x (3 - 1) = 3 x 1 x 2 = 6 ounces.