Puzzle of the Week
Self-Describing Numbers – 1

The number 1210 is a *Self-Describing Number* because each digit in order describes how many digits of that type there are - there is 1 0, 2 1's, 1 2, and 0 3's. SImilarly, 2020 is a Self-Describing Number because it has 2 0's, 0 1's, 2 2's, and 0 3's.

1	2	1	0
# of 0's	# of 1's	# of 2's	# of 3's

2	0	2	0
# of 0's	# of 1's	# of 2's	# of 3's

THE CHALLENGE: Find a Self-Describing Number that has five digits.

# of 0's	# of 1's	# of 2's	# of 3's	# of 4's

EXPLORATION: Why are there no Self-Describing Numbers with 1, 2, or 3 digits?

Puzzle of the Week

Self-Describing Numbers – 1 – Notes

THE CHALLENGE: A playful, disorganized approach to this is fine and should be encouraged.

Here are a few analytical ideas for attacking this puzzle.

Result 1: The sum of the digits is the number of digits. The digits count the digits of each type, so the sum of the digits counts all the digits.

Result 2: The sum of the products is the number of digits. Put another way, if you take the sum of multiplying each digit by the size of the digits it is counting, that sum must be the number of digits. This result follows from Result 1 because the sum of the products counts up all the digits involved in the number.

Look at the two given examples. In 1210, the sum of the products is (1 x 0) + (2 x 1) + (1 x 2) + (0 x 3) = 4. In 2020, the sum is (2 x 0) + (0 x 1) + (2 x 2) + (0 x 3) = 4.

Result 3: The rightmost, low-order digit, the ones digit, is 0. From Result 2, this digit must be 0 or 1 (or the product would be too large). Suppose it is 1. To avoid using variables, let's suppose we have a 6-digit number. Then the ones place counts the number of 5's. If there is a 5 someplace, then, by result 1, the only other non-zero digit must be a 1. But then we would have four 0's, which is impossible.

Result 4: The high-order digit is not 0. This must be true for this number to be considered a number.

Okay, let's construct some answers for 5-digit numbers.

There is a basic tension between results 1 and 2. To keep those sums the same, there must be an increase in 0's. For example, for each 2 there must be an extra 0, for each 3 there must be two extra 0's, for each 4 there must be three extra 0's, and so on. That's why most self-describing numbers have a lot of 0's.

If the number of 0's were 1, there would need to be nonzero numbers in every place except the far right place. But having all those nonzero numbers would necessitate having more 0's to make Result 2 work out. So, having only one zero in numbers with more than four digits is impossible. We can call that Result 5 if you like.

If there are 2 0's, the number is either 22100 or 21200. Of those, only 21200 works, and **that is the answer!**

If there are 3 0's, the number would be 32000, which doesn't work.

EXPLORATION: Let's look at the cases individually. Remember the four results when looking at these.

1 digit: Results 3 and 4 make this impossible.

2 digit: The number would have to be 20, which is not self describing.

3 digit: The number would have to be either 300, 210, or 120. None of these is self describing.