

Puzzle of the Week

Self-Describing Numbers – 2

The number 1210 is a *Self-Describing Number* because each digit in order describes how many digits of that type there are - there is 1 0, 2 1's, 1 2, and 0 3's. Similarly, 2020 is a Self-Describing Number because it has 2 0's, 0 1's, 2 2's, and 0 3's.

1	2	1	0
# of 0's	# of 1's	# of 2's	# of 3's

2	0	2	0
# of 0's	# of 1's	# of 2's	# of 3's

THE CHALLENGE: Find a Self-Describing Number that has seven digits.

# of 0's	# of 1's	# of 2's	# of 3's	# of 4's	# of 5's	# of 6's
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EXPLORATION: Why are there no Self-Describing Numbers with 6 digits?

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Self-Describing Numbers – 2 – Notes

THE CHALLENGE & EXPLORATION: A playful, disorganized approach to this is fine and should be encouraged.

Here are some useful results from the Notes on Self-Describing Numbers – 1:

Result 1: The sum of the digits is the number of digits.

Result 2: The sum of the products is the number of digits.

Result 3: The rightmost, low-order digit, the ones digit, is 0.

Result 4 & 5: The high-order digit is at least 2 for numbers with at least five digits.

There is a basic tension between results 1 and 2. To keep those sums the same, there must be an increase in 0's. For example, for each 2 there must be an extra 0, for each 3 there must be two extra 0's, for each 4 there must be three extra 0's, and so on. That's why most self-describing numbers have a lot of 0's.

Suppose the high-order digit is 2, so there are only two 0's. For a five-digit number, that means there are three nonzero digits. In Self-Describing Numbers – 1 we found that the only solution was 21200. For a six-digit number, there would be four nonzero digits. Also, since the digits have to add up to six, that would force the number to have two 1's and two 2's. None of the ways of mixing two 1's and two 2's work. The same thing keeps happening for longer numbers that have two 0's. This gives:

Result 6: The high order digit is at least 3 for numbers with at least six digits.

Let's keep playing with the number of 0's. What would be too many? Suppose the number of 0's is within three of the total number of digits. That would leave at most two more nonzero digits. One of those would be a 1 for the large 0 count. The only other possible nonzero entry would have to be for the number of 1's, which is not going to work out. We get yet another result:

Result 7: For numbers with at least six digits, the number of nonzero entries must be more than 3.

If you combine results 6 and 7, you will see that there cannot be any solutions for six-digit numbers. It is surprising that this works for Self-Describing numbers that have 5 or 7 digits, but not for ones that have 6 digits.

For 7-digit numbers, results 6 and 7 together guarantee that, if there is a solution, the number of 0's is 3. The other entries in the number must total 4. A few quick experiments quickly leads to the answer.

The answer is 3211000!