

Puzzle of the Week

Squares Made of Primes

THE CHALLENGE: For numbers that are squares bigger than 1, some squares are the sum of two prime numbers. For example, $4 = 2 + 2$ and $9 = 2 + 7$. Is there a square bigger than 1 that is not the sum of two prime numbers? If so, what is the smallest square, bigger than 1, that is not a sum of two primes?

$$n^2 = p + q?$$

EXPLORATION: What are some observations that can make your search more efficient? What happens if you are allowed to use more than two primes?

Puzzle of the Week

Squares Made of Primes – Notes

THE CHALLENGE: Goldbach's Conjecture says that all even numbers larger than 2 are the sum of two primes. This conjecture is unproven, but no one has found a counterexample, and people have searched to very large numbers (all numbers up to 4×10^{18}). So, all even squares are probably the sum of two primes, and we should check the odd squares.

To have two primes add up to an odd number, one of the primes must be 2. This makes checking very easy.

$$9 = 2 + 7$$

$$25 = 2 + 23$$

$$49 = 2 + 47$$

$$81 = 2 + 79$$

$$121 = 2 + 119 \text{ and } 119 = 7 \times 17$$

EXPLORATION: It has been proven that every even number starting with 4 is the sum of at most four primes. By adding "3" to that result, we know that every number is the sum of at most five primes. Someone thinks they have proven that every odd number is the sum of three primes.

So, it seems very likely that every even square is the sum of two primes and every odd square is the sum of either two or three primes, depending on whether $n^2 - 2$ is a prime. For example, $121 = 3 + 5 + 113$.