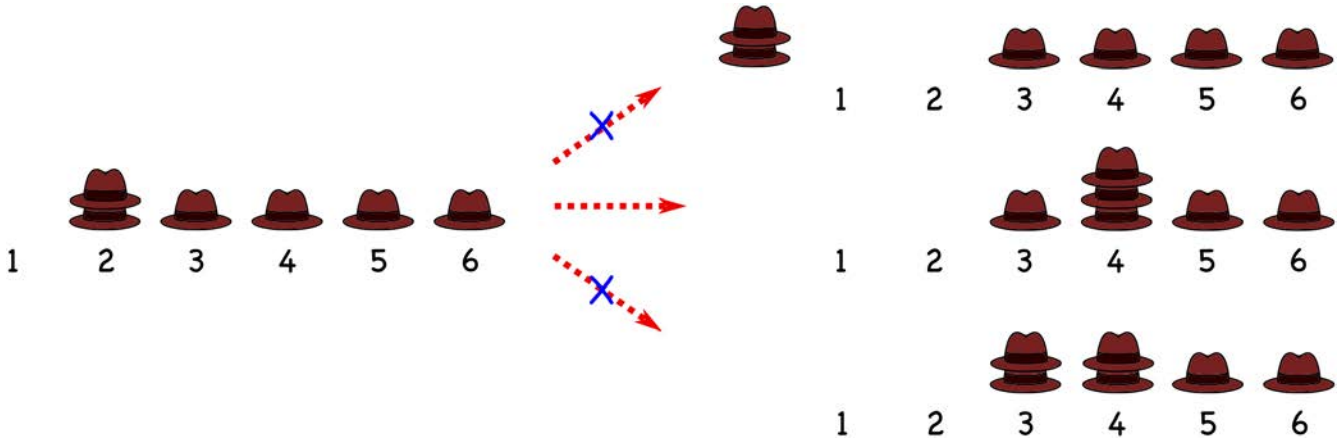
Puzzle of the Week
Stacking Hats – 2

Rules for stacking:

1) When you move a stack, you must move the whole stack onto a place with at least one hat.

2) A stack moves over the number of places for how many hats there are.

3) You can only use the original six spots.



THE CHALLENGE: Use these rules to move the six hats into one stack. The small blue hat needs to end up on top of the stack. Can the final stack of six hats end up in any of the six positions, or do only some of the positions work? Can the blue hat start in any position, or do only some of the starting positions work?

EXPLORATION: What happens if you start with seven hats in seven places? What changes if you allow hats to move to empty positions?

Puzzle of the Week
Stacking Hats – 2 – Notes

THE CHALLENGE & EXPLORATION: Dealing with a blue hat makes the Stacking Hats puzzle much trickier! If you look at the results below, you'll notice some clear patterns. However, I'll just describe what happens with different numbers of hats and leave it at that.

A couple general thoughts first. Because it is not legal to move a stack into an empty position, and the Blue hat must be moved away from its initial position so that it will be on top, it is never possible to have the final stack end up where the Blue hat started. To save work, I will only consider Blue hat positions for the positions on the left side - the other positions can be easily analyzed by taking mirror images of the left side positions.

The shorthand I will use is BL for Blue, BR for Brown, Y for yes, and N for no. For example, (BL BR BR) - (N Y N) means that when there are three hats ordered Blue - Brown - Blue, it is possible to have a stack of all three hats with the Blue hat on top only ending in the middle position.

1 Hat: (BL) - (Y)
2 Hats: (BL BR) - (N Y)
3 Hats: (BL BR BR) - (N Y N); (BR BL BR) - (Y N Y)
4 Hats: (BL BR BR BR) - (N Y N Y); (BR BL BR BR) - (Y N Y Y)
5 Hats: (BL BR BR BR BR) - (N Y N Y Y); (BR BL BR BR BR) - (Y N Y Y Y); (BR BR BL BR BR) - (Y Y N Y Y)
6 Hats: (BL BR BR BR BR BR) - (N Y N Y Y Y); (BR BL BR BR BR BR) - (Y N Y Y Y Y);
(BR BR BL BR BR BR) - (Y Y N Y Y Y)
7 Hats: (BL BR BR BR BR BR BR) - (N Y N Y Y Y Y); (BR BL BR BR BR BR BR) - (Y N Y Y Y Y Y);
(BR BR BL BR BR BR BR) - (Y Y N Y Y Y Y); (BR BR BR BL BR BR BR) - (Y Y Y N Y Y Y)

Allowing stacks of hats to jump into empty spots makes it possible to almost always succeed. I have put in red Y's for the new places that are possible.

1 Hat: (BL) - (Y)
2 Hats: (BL BR) - (N Y)
3 Hats: (BL BR BR) - (**Y** Y **Y**); (BR BL BR) - (Y N Y)
4 Hats: (BL BR BR BR) - (**Y** Y **Y** Y); (BR BL BR BR) - (Y **Y** Y Y)
5 Hats: (BL BR BR BR BR) - (**Y** Y **Y** Y Y); (BR BL BR BR BR) - (Y **Y** Y Y Y); (BR BR BL BR BR) - (Y Y **Y** Y Y)
6 Hats: (BL BR BR BR BR BR) - (**Y** Y **Y** Y Y Y); (BR BL BR BR BR BR) - (Y **Y** Y Y Y Y);
(BR BR BL BR BR BR) - (Y Y **Y** Y Y Y)
7 Hats: (BL BR BR BR BR BR BR) - (**Y** Y **Y** Y Y Y Y); (BR BL BR BR BR BR BR) - (Y **Y** Y Y Y Y Y);
(BR BR BL BR BR BR BR) - (Y Y **Y** Y Y Y Y); (BR BR BR BL BR BR BR) - (Y Y Y **Y** Y Y Y)