Puzzle of the Week

Water Cups – 2

You have two unmarked water cups. One holds 9 ounces and the other 15 ounces. You also have a large supply of water. You can use these two cups to create amounts other than 9 ounces and 15 ounces. For example, create 6 ounces in the larger cup by filling the 15-ounce cup and then pouring 9 of its ounces into the smaller cup.

THE CHALLENGE: Find all the amounts that you can create using these two cups.

9 Ounce

15 Ounce

EXPLORATION: Investigate other pairs of water cups that involve two numbers with a common divisor greater than 1. What patterns do you notice?

Puzzle of the Week
Water Cups – 2 – Notes

THE CHALLENGE & EXPLORATION: The two numbers, 9 and 15, have a greatest common divisor of 3. Whatever the greatest common divisor is, you will only be able to produce answers that are a multiple of their greatest common divisor.

The easiest way to work with this problem is to imagine creating a new unit. In this case, let's call that unit ThreeOunce, and it will equal 3 ounces. So we have one cup that holds 3 ThreeOunces and the other holds 5 ThreeOunces. Now, the analysis proceeds exactly as it did in "Water Cups - 1" with a 3-unit and a 5-unit cup.

Fill the **5-unit** cup.

Pour **3 units** from the larger cup into the smaller cup, leaving **2 units** in the larger cup.

Empty the smaller cup, pour the 2 units into the smaller cup from the larger cup, refill the larger cup, and fill the smaller cup. That leaves **4 units** in the larger cup.

Empty the smaller cup and pour 3 units from the larger cup into the smaller cup. That leaves **1 unit** in the larger cup.

So, we have a method that produces 1 through 5 units, as expected.

This translates into being able to produce 3 ounces, 6 ounces, 9 ounces, 12 ounces, and 15 ounces.

In general, we will be able to produce every multiple of the greatest common divisor up to the size of the bigger cup.