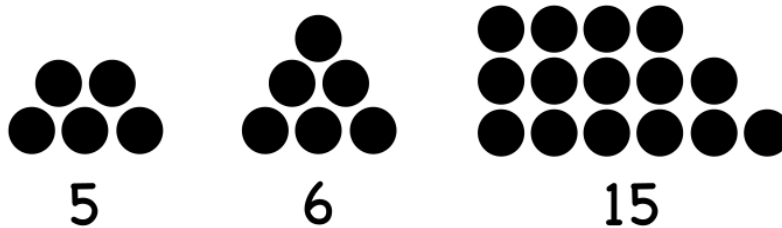


Puzzle of the Week

Trapezoidal Numbers – 2

Trapezoidal Numbers are the sum of two or more consecutive numbers. They deserve their name because you can make a trapezoid with that many dots, as pictured in the examples below. Note that having 1 dot on the top row is stretching the idea of being a trapezoid a bit, but it is allowed for these numbers.



THE CHALLENGE: 100 is a Square Number and a Trapezoidal Number. Find a group of consecutive numbers that add up to 100.

100 = 

EXPLORATION: Can you find other ways of adding up consecutive numbers to get a sum of 100?

Puzzle of the Week

Trapezoidal Numbers – 2 – Notes

THE CHALLENGE & EXPLORATION: Because consecutive numbers form an arithmetic sequence, the formula for the sum of k consecutive numbers starting at n is $k \times (n + (n + k - 1)) / 2$. You can think of this as k times the average value of all the numbers. You can also think of it as k times the median value of this arithmetic sequence of numbers.

If $100 = k \times (n + (n + k - 1)) / 2$, then $200 = k \times (2n + k - 1)$. Each way of factoring 200 as a smaller number times a larger number gives us a possibility to consider:

- 1×200 : $k = 1$, $n = 100$. We need at least 2 numbers, so $k = 1$ is impossible.
- 2×100 : $k = 2$, $n = 49 \frac{1}{2}$. n must be an integer, so this doesn't work.
- 4×50 : $k = 4$, $n = 23 \frac{1}{2}$. n must be an integer, so this doesn't work.
- 5×40 : $k = 5$, $n = 18$. $100 = 18 + 19 + 20 + 21 + 22$. Bingo!
- 8×25 : $k = 8$, $n = 9$. $100 = 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16$. Bingo!
- 10×20 : $k = 10$, $n = 5 \frac{1}{2}$. n must be an integer, so this doesn't work.

So, $100 = 18 + 19 + 20 + 21 + 22 = 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16$.